Examiners' Report
Principal Examiner Feedback

January 2021

Pearson Edexcel International Advanced Level
In Further Pure 2 (WFM02/01)
Paper 1: Further Pure F2

General

There was much excellent work seen but also scripts showed evidence of parts of the specification which had not been taught – almost certainly due to circumstances beyond the control of candidates and centres.

The use of the occasional diagram would help clarify what the objective was, especially in Q3 although there is no requirement to draw one. Some candidates may have used the graph from their graphical calculator without copying this into their solution. This is perfectly acceptable. Candidates would make it easier for themselves if they wrote down a standard formula before attempting to rearrange and use it. The M mark can then be awarded even if a slip is made when substituting values.

Many solutions are still not providing all the necessary details for full marks in questions of the "show that" type.

Report on individual questions

Question 1

Most candidates found this difficult and very few correct solutions were seen. The theory behind the question was not well understood. Candidates who substituted $w = z = (1 + \sqrt{3})$i immediately into the equation generally made good progress to a solution for p. Many solutions started by inserting $w = u + iv$ and $z = x+ iy$ and then aimed to find a real denominator for the fraction on the right-hand side. Little progress was made as candidates then failed to insert $u = x = 0$ and $v = y = (1 + \sqrt{3})$i. Some started by making z or p the subject of the formula though little progress resulted.

Question 2

This question was generally well answered with many candidates scoring full marks.

The addition of the fractions in part(a) was generally successful though a mark was occasionally lost through the misuse of brackets. The most common error was to start with the expression $\dfrac{(r+2)^2 - r(r+3)}{r(r+1)(r+2)}$. To gain both marks it was essential either to include the two original fractions or state "LHS = "...

Many candidates reached the correct answer in part (b) with only a few slips in the algebra seen. Candidates who had difficulty starting this part failed to write down a list of a few terms to be added and then split them up into the difference of terms as indicated by the use of "hence" on the question paper. Once written out it became clear that only two terms remained to combine. A minority decided to use a three partial fraction approach and achieved a correct final expression. This scored zero as it was the wrong method.

Question 3

A diagram in which the two graphs $y = |x^2 + x - 2|$ and $y = (x + 5)/2$ were superimposed would have helped many candidates. This can be copied from a graphical calculator or use could be made of the diagram without copying it into the work. Most candidates progressed from $x^2 + x - 2 < (x + 5)/2$ to the solution of a quadratic equation; some errors occurred in the algebraic rearrangement and a mark was occasionally lost as a wrong answer to the candidate's equation was given when no correct quadratic formula had been written down.

Candidates who wrote down $-x^2 - x + 2 < (x + 5)/2$ found the quadratic equation easy to solve though again there were algebraic errors. Occasionally this led to complex root being seen.

Solving $x^2 + x - 2 = 0$ was often seen and the two answers from this confused the thinking for the conclusion. A limited number of candidates managed to use the four critical values to write down two correct equalities.

Question 4

Many excellent solutions were seen to this question though careless errors caused marks to be lost. A few blank responses were submitted. A few candidates failed to differentiate the formula $y^2 = z^{-1}$ correctly and so no progress was possible in part (a). The most common approach was to achieve $dy/dz = -1/2\, z^{-3/2} dz/dx$ though a few preferred to use $dy/dx = -1/2\, y^3 dz/dx$. Generally a correct derivative led to the result $dz/dx - 4z = -6x$ though sign errors were seen even though the answer was given.

In part (b) most candidates found the integrating factor e^{-4x} and multiplied the equation throughout by it. I.F. $= e^{+4x}$ was seen a number of times. The left hand side occasionally appeared as $d/dx(y\, e^{-4x})$ which lost 3 marks.

Integration by parts was generally well done though $\int e^{-4x}\, dx = -4e^{-4x}$ appeared a number of times. A number of solutions did $\int x\, e^{-4x}\, dx$ correctly but then failed to multiply by "-6" - an expensive error. A common error was to write $z = 3x/2 + 3/8$. The omission of a constant of integration cost two accuracy marks.

Some solutions preferred to solve the differential equation using a C.F./ P.I. approach. The most common error here was to write the C.F. as $y = (Ax + B) e^{-4x}$ as a result of only having a single value $m = -4$.

Part (c) was an easy mark for realising the need to write y^2 as the reciprocal of the answer for (b). A number of solutions went from $z = 3x/2 + 3/8 + Ae^{4x}$ to $y^2 = 1/(3x/2) + 1/(3/8) + 1/(Ae^{4x})$.

Question 5

Most candidates were able to score some marks on this question and many complete solutions were seen.

The differentiation of the given equation in part (a) to find d^3y/dx^3 was accessible to most candidates. The easiest method was a term by term approach and the differentiation of $(2 - x^2)d^2y/dx^2$ and $3y$ were generally accurate. The derivative of $5x(dy/dx)^2$ was often seen as $5(dy/dx)^2 + 10x \, dy/dx$ with an immediate loss of three marks. A few candidates rewrote the formula as $d^2y/dx^2 = (3y - 5x(dy/dx)^2))/(2-x^2)$ and attempted quotient rule differentiation; this proved challenging. Again there were chain rule errors and substitution for $3y$ was often not attempted. Few successfully reached the printed answer.

In part(b) evaluation of the derivatives d^2y/dx^2 and d^3y/dx^3 was generally successful though some careless errors were seen e.g. using both 9/2 and ½ in the d^3y/dx^3 formula.

A correct general Taylor Series formula written down would have made marking easier when errors occurred. Most solutions included the factorial terms though a constant of 1 or zero occasionally appeared. Many solutions, including otherwise correct answers, gave a final answer as f(x) = …when y = …was expected.

Question 6

Many fully correct solutions were seen.

In part (a) the solution of the auxiliary equation was generally correct though 1 +/- 2i and -1 +/-4i were seen. The preferred form of the C.F. was $y = e^{-x}(A\cos2x + B\sin2x)$. Candidates using $Ae^{(-1+2i)x} + Be^{(-1-2i)x}$ generally encountered difficulties when solving for A and B in part (b). One or two solutions of the form $e^{-1}(A\cos2x + B\sin2x)$ were seen. A correct form of the P.I. $y = a\cos x + b\sin x$ was generally used though $y = a\cos x$ or $y = b\sin x$ were sometimes seen. Differentiating twice and substituting generally led to two simultaneous equations though a

number of careless errors were seen when solving them. The final mark for the general solution was often lost as candidates wrote "GS =   " or "General solution =   " rather than

$y = e^{-x}(A\cos 2x + B\sin 2x) + 6/5\cos x + 3/5\sin x.$

In part (b) the use of $x = 0$ and $y = 0$ to find $A = -6/5$ was generally well done though $A = -5/6$ was seen. Most attempted a product rule differentiation though there were a number of sign errors, careless slips and chain rule errors. A common result was $B = 3/10$ rather than $-9/10$. A lack of "$y =$" often lost the final accuracy mark.

Question 7

Most candidates realised that the method required in part (a) to find where the tangent was perpendicular to the initial line was to write down $x = r\cos\theta$, differentiate and set $dx/d\theta = 0$. A few solutions used $y = r\sin\theta$ and lost a minimum of three marks. Equally popular methods were the product rule differentiation of $x = 3\sin 2\theta\cos\theta$ or $x = 6\sin\theta\cos^2\theta$ before solving $dx/d\theta = 0$. Use of a double angle formula appeared in various places leading to a solution of either $6 - 18\sin^2\theta = 0$, $18\cos^2\theta - 12 = 0$ or $6 - 3\tan 2\theta\tan\theta = 0$. The algebra then used to reach $\tan\phi = \frac{1}{2}$ was not always convincing.

In part (b) a few solutions tried to calculate ϕ in radians but this was never going to give an exact value for R. Most candidates managed to calculate exact values for $\sin\phi$ and $\cos\phi$ and hence $R = 2\sqrt{2}$. A few solutions just wrote down a correct answer.

Most solutions for part (c) wrote down a correct formula for the area of the sector though many then failed to rewrite $\sin^2 2\theta$ as $(1 - \cos 4\theta)/2$. Expanding $\sin^2 2\theta$ as $4\sin^2\theta\cos^2\theta$ was quite common though there was seldom progress. Correct integration of $1 - \cos 4\theta$ was common though $\theta + 4\sin\theta$ was seen. A limited number of responses reached the point where it was necessary to evaluate $\sin 4(\arctan(1/\sqrt{2}))$. Some solutions used a calculator to reach $4\sqrt{2}/9$ whereas others followed a more traditional route of evaluating $2\sin 2\phi \cos 2\phi$ using their answers from part (a). A final answer of $9\arctan(1/\sqrt{2})/4 - \sqrt{2}/4$ was not seen very often.

Question 8

(a) Progress from $z = e^{i\theta}$ to $z^n + 1/z^n = 2\cos n\theta$ produced a disappointingly number of fully correct solutions. $z^n = \cos n\theta + i\sin n\theta$ was nearly always correct. $1/z^n = \cos n\theta - i\sin n\theta$ was common but often the step $1/z^n = \cos(-n\theta) + i\sin(-n\theta)$ was omitted. Solutions which did this correctly often failed to show $z^n + 1/z^n$ as a sum of four terms before writing down $2\cos n\theta$. A few solutions only wrote down a result using $n = 1$.

(b) A binomial expansion of $(z + 1/z)^6$ was generally written down correctly and, with the use of pairing of terms and part (a), a formula $2\cos6\theta + 12\cos4\theta + 30\cos2\theta + 20$ was reached. Occasionally it was unclear that $(z + 1/z)^6 = 64\cos^6\theta$ or why 1/32 appeared in the final answer. One or two responses had the expansion of $(\cos\theta + i\sin\theta)^6$ as the starting point and, using real parts, a correct formula for $\cos6\theta$ was achieved. Few attempts then managed to find $\cos^4\theta$ as a sum of $\cos4\theta$ and $\cos2\theta$ terms. It was pleasing to see a few solutions get the correct result; it was hard work though!

Most solutions progressed from the printed equation in part (c) to $32\cos^6\theta = 10$. Disappointingly few achieved two correct values for θ. 0.603 was a common correct value though a second solution of 2.54 was generally missing or written as 2.539. Several solutions had $\theta = 0.824$ when in fact $\cos\theta = 0.824$.

There were many excellent solutions to $\int(32\cos^6\theta - 4\cos^2\theta)\, d\theta$ in part (d). A few solutions failed to use the earlier result and made no progress. Likely errors were an incorrect formula for $\cos^2\theta$, a bracketing error with the multiple of 4 or differentiation rather than integration of the trigonometric functions.